

# Equivalent dynamic model of multilayered structures with imperfect interfaces: application to a three-layer plate with sliding interfaces

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*This work aims to build an equivalent model of multilayered structures with imperfect interfaces for vibro-acoustic characterization. To do so, new boundary conditions containing the imperfections are implemented. The dispersion relation is derived from the equivalent model and the bending wavenumber is obtained. In this work we applied the methodology to a sandwich-structured composite with sliding interfaces. We computed the flexural rigidity and the damping of the flexural waves to see their effects.*

## 1 Context and objective

This work aims to develop an equivalent model able to characterize and optimize the behavior of multilayered structures with imperfect interfaces and particularly a sandwich-structured composite, mainly used in the industry because of its high ratio stiffness over mass. Equivalent plate models are increasingly developed in the scientific community these last years [1, 2, 3, 4], because of their several advantages. For instance: a minimum calculation time allows the testing of many configurations efficiently *i.e.* without using a finite element model mainly 3D with a significant mesh; an easy-to-use method; physical meaning brought by the physical model. The goal of this work is to extend the latest improvement of the Guyader's model [5] done by Marchetti [6] for composite plates, by taking into account imperfect interfaces, and more specifically as a first step sliding between the layers.

## 2 Methodology

Previously, in Guyader's and Marchetti's work, continuity conditions were formulated for transversal shear stresses and the 3D displacement field. These conditions yield the expression of the cinematic variables of a layer in terms of the previous one, thanks to a coupling matrix. Finally, the global behavior of the composite is described by using only 5 cinematic variables of the reference layer. Thus, to take into account imperfect interfaces with non-canonical coupling conditions, we modified the continuity conditions of the displacements into conditions introducing imperfect interfaces, such as sliding or delaminations alike Massàbo's work [7]. An interface is modeled as a zero thickness layer, which contains local perturbations of the layer  $n$  written as jumps of displacement symbolized by

$$\hat{u}^n = u^n - u^{n-1}. \quad (1)$$

Therefore, a constitutive equation of the interfaces is added to compute the sliding displacement  $\hat{u}^n$  with respect to the shear stresses applied at the interface

$$(\hat{\sigma}^n) = [K^n].(\hat{u}^n) + (t^n), \quad (2)$$

where the interface stresses  $\hat{\sigma}^n$  is written as a piecewise linear function in order to be able to model many different types of interfaces, such as elastical or dissipative.  $K^n$  is the stiffness of the interface  $n$  and  $t^n$  the dissipation. The displacements, the interface stress and interface stiffness are illustrated in Fig. (1a). Once the displacement field of the reference layer is computed, the principle of least action is applied and leads to a system of differential equations, equivalent to a mass matrix and a stiffness matrix. Solving the determinant of the equivalent matrix yields the different wavenumbers according to the different types of movements. They are sorted to retrieve only the solutions corresponding to the bending wavenumber. From now, several equivalent parameters can be evaluated, such as the equivalent flexural rigidity, or the equivalent damping (loss factor) for further analysis.

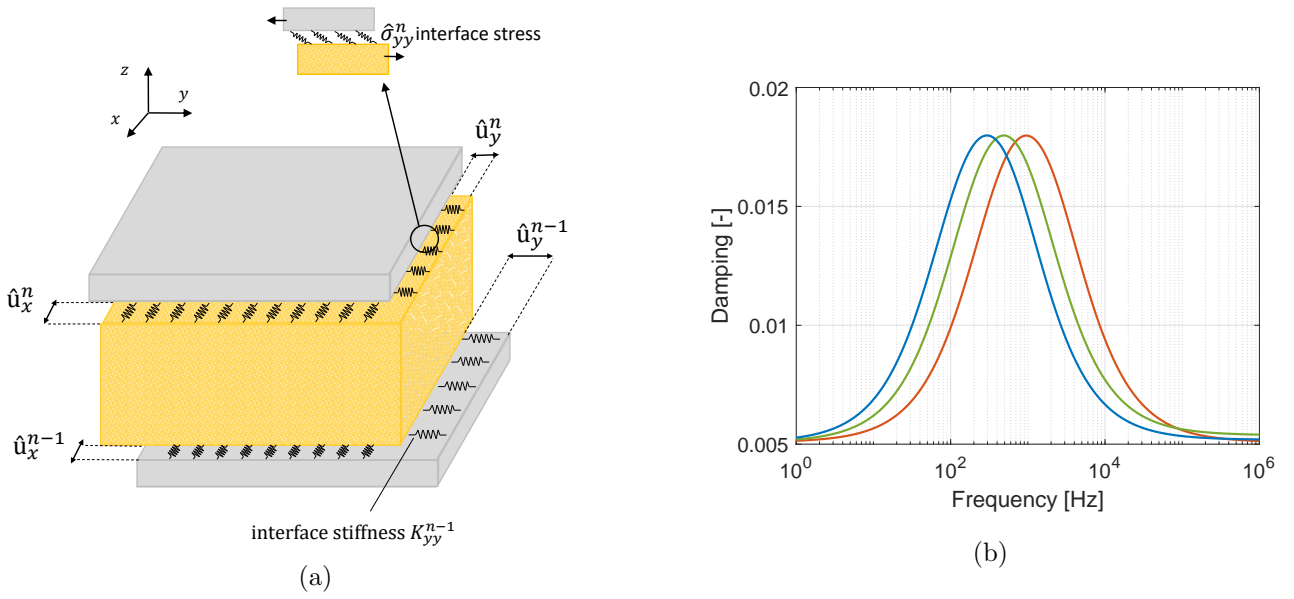


Figure 1: (a) Typical three-layered composite with a soft and thick core and illustration of the sliding with the generated interface stiffness, (b) Equivalent loss factor with perfect interfaces (—) and with imperfect interfaces: one imperfect interface (—), two imperfect interfaces (—).

### 3 Results

We show in this work the impact of sliding interfaces on the equivalent flexural rigidity and damping of a typical three-layered composite with a thick and soft core (Fig. 1b). The impact is mainly a shift of the frequency of optimum damping towards the low frequencies, as well as a shift of the transition zone for the flexural rigidity, which is governed by the shearing of the core. The behavior of the composite stay unchanged in the low frequencies. For the high frequencies, the contribution of the sliding is not important enough to be physically valuable. Other effects and the asymptotic behaviors will be investigated and presented during the communication.

### References

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