Low frequency acoustic method to measure static thermal permeability

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1 Introduction

Acoustic behaviour of air-saturated rigid-frame porous material is described by equivalent fluid model through complex density $\tilde{\rho}$ and bulk modulus $\widetilde{K}$. Their experimental characterization can be made in impedance tube through 3-microphone or 4-microphone techniques. From an analytical point of view, semi-phenomenological models are commonly used to describe the sound propagation inside solid matrix filled by a fluid medium in terms of equivalent fluid [1-3]. The accuracy of each model is strictly linked to the number of input parameters which describe thermal and viscous dissipation phenomena inside the core. These so-called transport parameters can be evaluated through three different methods: an indirect one, where parameters are obtained from the analytical expression of complex density and bulk modulus, an inverse one, based on optimization procedure to match acoustic measurements and semi-phenomenological model, and a direct one, based on their definitions. In particular, to evaluate experimentally static viscous permeability k_0 (equivalent to static air flow resistivity σ) and static thermal permeability k_0' by using their definitions, low frequency measures are requested. In literature, an experimental alternative method to calculate static viscous permeability k_0 has been proposed [4]. In this paper, the aim is to extend this methodology for the calculation of static thermal permeability k_0'.

2 Theoretical formulation

The static viscous permeability k_0 can be derived as the low frequency limit of dynamic permeability $\tilde{k}(\omega)$. Combining Darcy's law with solution of linearized equation of motion, it can be written

$$k_0 = \lim_{\omega\to 0} \tilde{k}(\omega) = \lim_{\omega\to 0} \frac{\delta_\nu}{2i} \frac{\rho_0}{\tilde{\rho}_{eq}} \tag{1}$$

where $\delta_\nu = \sqrt{2\mu/\omega\rho_0}$ is the viscous penetration depth, ρ_0 the fluid density and $\tilde{\rho}_{eq}$ the equivalent complex density of porous medium. A thermal analogue of Darcy's law is defined by Lafarge, where a static thermal permeability $k_0' = \lim_{\omega\to 0} \tilde{k}'(\omega)$ is introduced to improve the description of low frequency behaviour of dynamic bulk modulus $\widetilde{K}$. Operating the limit for the low frequency, static thermal permeability results from solution of linearized equation of energy and Thermal Darcy's law

$$k_0' = \lim_{\omega\to 0} \tilde{k}'(\omega) = \lim_{\omega\to 0} \frac{\gamma}{\gamma-1} \frac{\delta_\kappa}{2i} \frac{p_m}{\widetilde{K}_{eq}} \tag{2}$$

with $\delta_\kappa = \sqrt{2k/\omega\rho_0 c_p}$ the thermal penetration depth, p_m the atmospheric pressure and $\widetilde{K}_{eq} = \varphi\widetilde{K}$ is the equivalent complex bulk modulus.

3 Static thermal permeability measurement

Starting from the test configuration suggested by Dragonetti et al. to measure the air-flow resistivity through "indirect acoustic method" (Figure 1.a), the aim of this work is to derive an acoustic methodology for the estimation of static thermal permeability k_0'.Hayden and Swift [5] proposed an experimental methodology to evaluate thermal Rott's function f_κ, which is linked to complex bulk modulus $\widetilde{K} = \gamma p_m/(1+(\gamma-1)f_\kappa)$. Their apparatus was made of an oscillating piston which produced changes in the volume of a cylindrical chamber. This method required an upscale of the geometry of the porous material, the estimation of pressure beyond the tested sample and the volume perturbation with and without the material, respectively $(V_1, p_1)_{full}$ and $(V_1, p_1)_{empty}$, (see Figure 1.b and Figure 1.c):

$$f_\kappa = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_m}{V_{tot}\varphi}\left[\left(\frac{V_1}{p_1}\right)_{full} - \left(\frac{V_1}{p_1}\right)_{empty}\right] + \frac{1}{\gamma-1}\frac{1-\varphi}{\varphi} \tag{3}$$

In the present work, the mechanical input of Hayden's apparatus (oscillating piston) is substituted with an acoustic driver placed between two small cavities and the porous material is not up scaled. For wavelengths much larger than the maximum linear dimension of the cavities, the volume perturbation V_1 is linked to the volume velocity U as follow:

$$V_1 = C_{dw} p_{dw} \quad (4)$$

where p_{dw} is the sound pressure measured in the lower cavity, and $C_{dw} = V_{dw}/\gamma P_0$ is the acoustic compliance of the air in the lower cavity and V_{dw} is the compressible air-volume in the lower cavity.

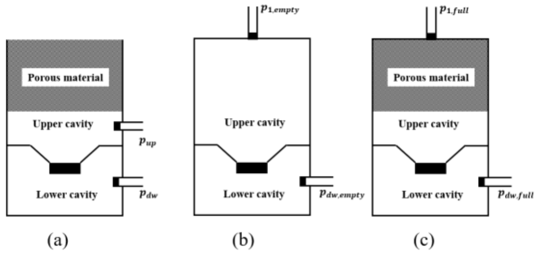

Figure 1. (a) Dragonetti et al. set up for air-flow resistivity measure. (b) "empty" and (c) "full" acoustic set up of Hayden's apparatus.

4 Results and conclusion

Numerical procedure has been used to aim the reliability of the proposed methodology. Different samples have been simulated by fixing their transport parameters. Therefore, dynamic thermal permeability, as well as static thermal one, is obtained combining Eq. (3), (4) in (2). In Figure 2, it can be seen that in the low frequency limit of dynamic thermal permeability, the imaginary part tends to zero value, while the real part tends to the fixed value for the tested samples.

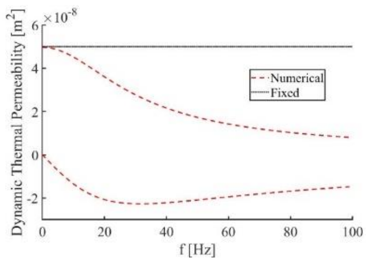

Figure 2. Dynamic thermal permeability: (--) dotted line is for numerical simulation, (-) continuous line is the fixed value.

Numerical simulations are carried out by varying the lower cavity volume and for different materials. In the limit of assuming "lumped parameters", results shows that measures are not affected by V_{dw}. In the table above, estimating percentual errors versus the increasing frequency are reported for different materials. Future experimental tests will be carried out to establish how thermal dissipation in tube can affect the measures and their repeatability.

***k0'* fixed [m²]**	3.41E-08		1.4411E-08		7.2046E-09	
***f* [Hz]**	***k0'* evaluated [m²]**	**%error**	***k0'* evaluated [m²]**	**%error**	***k0'* evaluated [m²]**	**%error**
2	3.418E-08	-0.34	1.446E-08	-0.31	7.219E-09	-0.20
5	3.408E-08	-0.03	1.445E-08	-0.25	7.218E-09	-0.18
10	3.370E-08	1.08	1.441E-08	-0.01	7.213E-09	-0.12
15	3.309E-08	2.87	1.436E-08	0.37	7.205E-09	-0.01
20	3.227E-08	5.27	1.428E-08	0.90	7.194E-09	0.15

Table 1. Fixed and evaluated data for thermal permeability.

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