Analytical approximations for sound absorption by labyrinthine architectures

K. Attenborough

Engineering and Innovation, The Open University,
Milton Keynes, MK7 6AA, UK

1 Introduction

Predictions and measurements have shown that labyrinthine architectures with long convoluted internal sound channels providing ‘coiled up space’ give good impedance matching [1] and yield high absorption coefficients at wavelengths much greater than the layer thickness [2]. Most computations for labyrinthine structures have used numerical methods such as FEM. But an alternative is to regard an array of narrow labyrinthine channels in a non-porous solid to be a porous material with a high tortuosity. In this paper, contiguous units each of which contains a labyrinthine slit are considered to form a rigid-porous solid acting, acoustically, as an effective fluid layer. Analytical approximations for the acoustical properties of the labyrinth make use of the classical result for sound propagation in tortuous slits [3]. Dual porosity theory is used to predict the results of making a labyrinthine perforation in a porous matrix having high flow resistivity and tortuosity. Example predictions for 4.5 cm thick, porous layers containing labyrinthine slit perforations, suggest the labyrinthine slit perforations in a high flow resistivity matrix have the potential for achieving narrow deep subwavelength absorption peaks. The presented approximations should be useful in designing labyrinthine metamaterials with targeted acoustical performance.

2 Labyrinthine slits parallel and normal to the surface

Consider a plane wave incident on a layer of contiguous rectangular units each of which contains a single labyrinthine slit of width (b) with N folds in the slit formed by walls of length l and width w. The walls are either parallel to (Figure 1(a)) or normal to (Figure 1(b)) the surface on which the wave is incident.

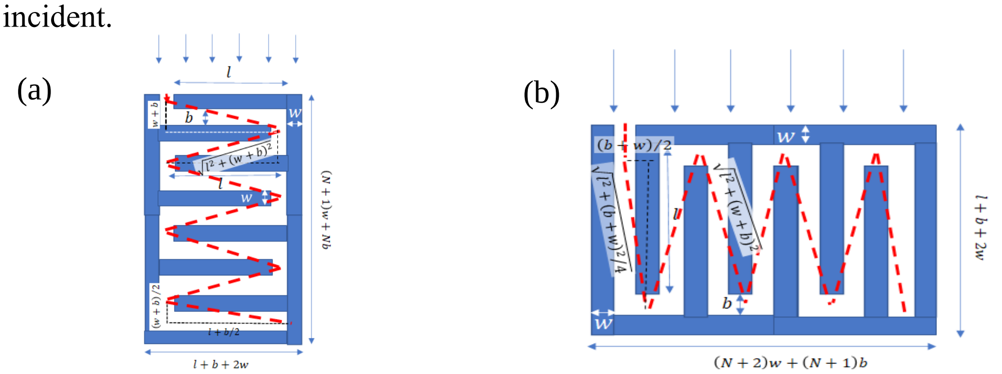

Figure 1: (a) Units containing a single labyrinthine slit, formed by walls either (a) parallel or (b) normal to the surface, showing the associated dimensions and approximate streamline paths.

The shortest path fluid streamline paths through the slit labyrinths are approximated by zig zags (red broken lines in Figures 1(a) and (b)). Expressions for tortuosity are respectively:

$$T_H=\left\{\frac{(N-1)\sqrt{(l+b)^2+(b+w)^2}+\sqrt{(l+b/2)^2+(b+w)^2/4}+w/2-((N+1)w+Nb)}{(N+1)w+Nb}\right\}^2,$$

$$T_V\left\{\frac{(N-1)\sqrt{l^2+(b+w)^2}+\sqrt{l^2+(b+w)^2/4}+\sqrt{(l+b/2)^2+(b+w)^2/4}+w+b/2-(l+b+2w)}{l+b+2w}\right\}^2 \quad (1)$$

The bulk, Ω_B, and surface, Ω_S, porosities of the labyrinth structures are, respectively,

$$\Omega_B=\{[b(Nl+w)+(N-1)(2b+w)]/[((N+1)w+Nb)(l+b+2w)]\},\ \Omega_S=b/(l+b+2w),$$

$$\Omega_B=\{[(N+1)(l+b+w)b]/[((N+2)w+(N+1)b)(l+b+2w)]\},\ \Omega_S=b/((N+2)w+(N+1)b) \quad (2)$$

The complex density, ($\rho_L(\omega)$), complex compressibility, ($C_L(\omega)$), propagation constant, ($k(\omega)$), and normalised impedance ($Z_L(\omega)$) of an effective fluid medium equivalent to such a labyrinth structure are, respectively,

$$\rho_L(\omega)=(T/\Omega_B)\rho(\omega),\ \rho(\omega)=\rho_0/H(\lambda),\ H(\lambda)=1-\tanh(\lambda\sqrt{-i})/(\lambda\sqrt{-i}),\ C_L(\omega)=\Omega_B C(\omega),$$

$$C(\omega)\ =\ (\rho_0 c_0^{2})^{-1}[\gamma-(\gamma-1)H(\lambda\sqrt{(N_{PR})})],\ \lambda=(b/2)\sqrt{\omega\rho_0/\mu},\ k_L(\omega)=\omega\sqrt{\rho_L(\omega)C_L(\omega)},$$

$$Z_L(\omega)=(\Omega_S\rho_0 c_0)^{-1}\sqrt{\rho_L(\omega)/C_L(\omega)}\,. \tag{3}$$

$\rho_0, c_0, N_{PR} \wedge \mu$ are density, sound speed, Prandtl number and dynamic viscosity in air, respectively.

3 Labyrinthine slit perforations in a rigid-porous matrix

With sufficient scale separation between matrix pore size and labyrinthine slit width, a perforated porous matrix may be regarded as a dual porosity system. The resulting complex density, ρ_{dpL}, and complex compressibility, C_{dpL}, are [3]

$$\rho_{dpL}(\omega)=\left[\frac{1}{\rho_L(\omega)}+\frac{(1-\Omega_B)}{\rho_m(\omega)}\right]^{-1},\ C_{dpL}(\omega)=C_L(\omega)+(1-\Omega_B)F_d(\omega)C_m(\omega), \tag{4}$$

where ρ_m and C_m represent the complex density and compressibility of the rigid-porous matrix and $F_d(\omega)$ is a pressure diffusion function.

Figures 2(a) and (b) show predictions of normal incidence absorption spectra for 5 cm thick hard-backed layers. If made in a non-porous rigid solid structure, labyrinthine slit perforations are predicted to yield small narrow absorption peaks (see the broken black lines in Figures 2(a) and (b)). But when labyrinths are made in a matrix with high flow resistivity and tortuosity, more useful low-frequency absorption peaks are predicted. Predictions in Figs. 2(a) and (b) are for a slanted slit matrix [4] perforated by an eight-fold horizontal slit labyrinth and a ten-fold vertical wall labyrinth respectively. The peaks at 260 Hz and 170 Hz in Figs. 2(a) and 2(b) correspond to the layer thickness being 1/26th and 1/40th of the incident wavelengths, respectively.

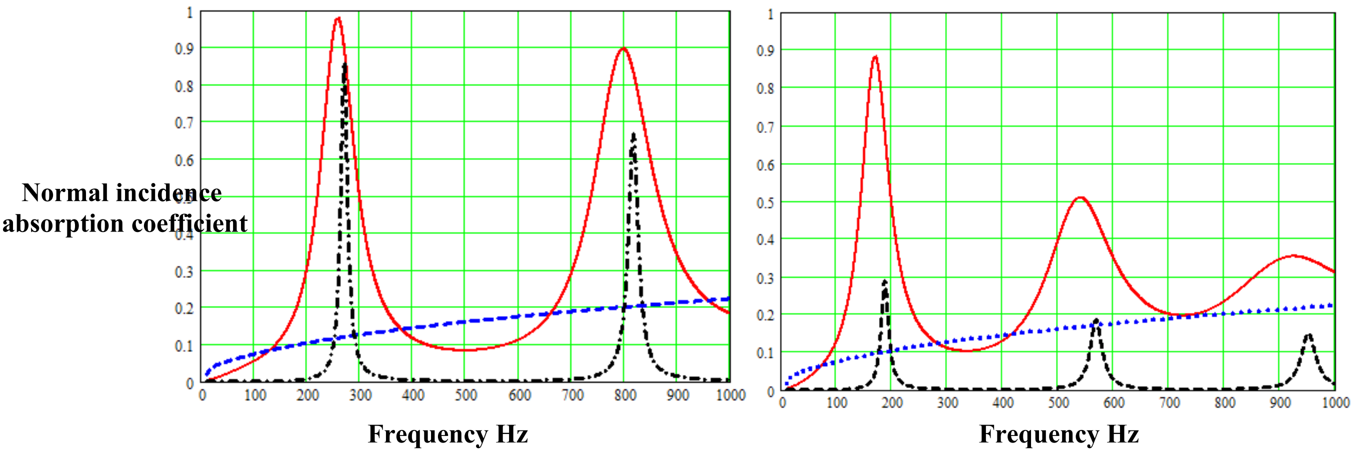

Figure 2: Normal incidence absorption coefficient spectra (red continuous lines) predicted for 0.05 m thick hard-backed layers consisting of (a) a horizontal wall labyrinth (w = 1 mm, b = 5.1 mm, l = 40 mm, N = 8) (b) a vertical wall labyrinth (w = 1 mm, b = 3 mm, l = 45 mm, N = 10) in an inclined slit medium (0.04 mm wide slits at 70° to the normal, porosity 0.9).
Black broken lines are predictions for corresponding labyrinthine perforations in a non-porous solid.
Blue dotted lines are predictions for each unperforated porous layer.

References

[1] W. Tang and C. Ren, Total transmission of airborne sound by impedance-matched ultra-thin metasurfaces, J. Phys. D: Appl. Phys. **50** 105102 (2017).

[2] A. Krushynska, *et al*, Labyrinthine Acoustic Metamaterials with Space-Coiling Channels for Low-Frequency Sound Control, Acta Acustica united with Acustica, **104** 200 – 210 (2018).

[3] X. Olny, C. Boutin, Acoustic wave propagation in double porosity media, J. Acoust. Soc. Am. **114** 73–89 (2003)

[4] K. Attenborough, Microstructures for lowering the quarter wavelength resonance frequency of a hard-backed rigid-porous layer, Applied Acoustics, **130** 188–194 (2018).