A vibration transmissibility dynamic poroelastic model for open cell polyurethane foam involving viscoelasticity and pneumatic damping effects

Qicheng Zhang[1], Fabrizio Scarpa[1*], David Barton[2], Yunpeng Zhu[3], Ziqiang Lang[3]

[1]Bristol Composites Institute (ACCIS), University of Bristol, BS8 1TR Bristol, UK
[2]Department of Engineering Mathematics, University of Bristol, BS8 1UB Bristol, UK
[3]Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK

This paper presents a dynamic poroelastic model for polyurethane (PU) foam considering the viscoelasticity of the PU material and the pneumatic damping of the porous structure. Stress relaxation, quasi-static cyclic loading and vibration transmissibility tests have been performed on PU foam samples to identify viscoelastic and permeability parameters.

1 Introduction

PU foam is widely used for vibration or noise absorption. The high damping of the foam is mainly caused by the viscoelasticity of the PU material and the pneumatic damping effect associated to the porosity and dynamic permeability. Bianchi and Scarpa [1] have previously developed dynamic models for PU conventional and auxetic foam considering only the pneumatic damping phenomenon. White et al [2] have also developed dynamic models of PU foam focusing on the viscoelasticity and ignoring the pneumatic damping effect. In this work we have modified the pneumatic damping model to also consider more realistic air flow perpendicular to the loading direction and including the intrinsic viscoelasticity of the polyurethane. Hereditary integral models based on Prony series [3] are widely used to describe viscoelasticity. We have here developed a method to consider the stress relaxation test and quasi-static cyclic loading test data simultaneously to overcome known identification problems of viscoelastics [2] and determine the Prony series parameters for the viscoelastic material based on a nonlinear regression method.

2 Methodology and results

The Reynolds number within the internal airflow in the open cell foam is estimated to be around 0.1 in our testing conditions, thus the linear Darcy's law can be applied. The air is assumed to be incompressible and the air flow along the direction of vibration is ignored because the layout of the vibration transmissibility rig [1]. The relation between the pneumatic damping force and deformation rate of the foam can be described in equation (1), where μ is the dynamic viscosity of the air, φ is porosity of the foam, K is permeability, W=30mm is width of the foam sample and H=15mm is height of the sample.

$$F_{a-z}\left(\dot{z}\right)=\frac{W^4\mu}{32\varphi KH}\dot{z} \tag{1}$$

Stress-time curves of the PU in stress relaxation and cyclic loading tests are shown in equation (2) and (3), obtained from hereditary integral model based on Prony series. Fourier series with order M=20 are used to describe the triangle wave during the cyclic loading. E_0 is modulus (linear within small strain ranges up to <2%). a_i and α_i are parameters of Prony series with order N=3.

$$\sigma_1\left(t\right)=E_0\varepsilon_0\left[1-\sum_{i=1}^{N}\frac{a_i}{\alpha_i}\left(1-e^{-\alpha_i t}\right)\right] \tag{2}$$

$$\begin{aligned}\sigma_2\left(t\right)=&\frac{8E_0\varepsilon_{\max}}{\pi^2}\sum_{k=0}^{M-1}\left[\frac{\left(-1\right)^{k+1}}{\left(2k+1\right)^2}\sin\left(\left(2k+1\right)\omega_0 t\right)\right]-\\&\frac{8E_0\varepsilon_{\max}}{\pi^2}\sum_{i=1}^{N}\sum_{k=0}^{M-1}\left\{\frac{\left(-1\right)^{k+1}\omega_0 a_i}{\left(2k+1\right)\left(\alpha_i^2+\left(2k+1\right)^2\omega_0^2\right)}\left[-\cos\left(\left(2k+1\right)\omega_0 t\right)+\frac{\alpha_i}{\left(2k+1\right)\omega_0}\sin\left(\left(2k+1\right)\omega_0 t\right)+e^{-\alpha_i t}\right]\right\}\end{aligned} \tag{3}$$

$$\chi^2\left(\mathbf{p}\right)=\sum_{i=1}^{N_1}\left[\sigma_{1i}-\sigma_1\left(t_{1i},\mathbf{p}\right)\right]^2+\beta\sum_{j=1}^{N_2}\left[\sigma_{2j}-\sigma_2\left(t_{2j},\mathbf{p}\right)\right]^2 \tag{4}$$

The error function is shown in equation (4), where σ_{1i}, t_{1i}, σ_{2j} and t_{2j} are the experimental data from the stress relaxation and cyclic loading tests, β=2.5 is the weight factor and **p** are viscoelastic parameters. The

Marquardt-Levenberg Method is used to minimize the error and obtain the viscoelastic parameters (Table 1). The comparison between experimental and simulated curves related to stress relaxation and cyclic loading tests (strain rate $\dot{\varepsilon}$=0.00056/s) are in Figure 1 (a) and (b), showing good agreement.

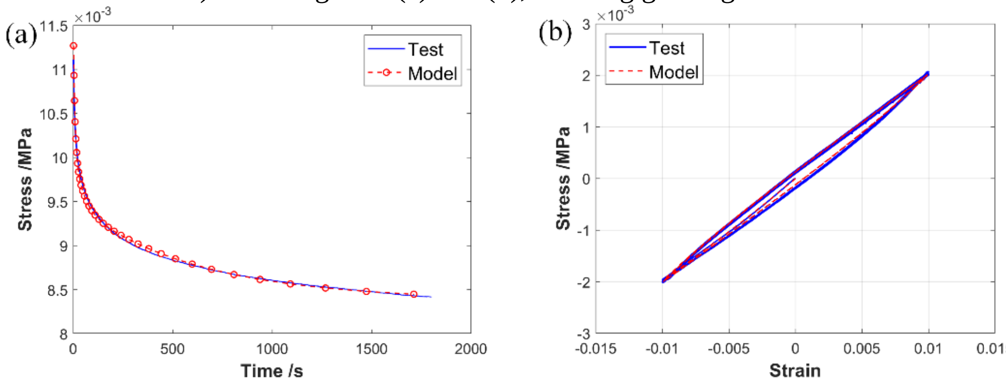

Figure 1 Stress relaxation test result (a) and quasi-static cyclic loading test result (b)

Symbol	E_0 /MPa	a_1 /s^{-1}	a_2 /s^{-1}	a_3 /s^{-1}	α_1 /s^{-1}	α_2 /s^{-1}	α_3 /s^{-1}
Value	0.218958	0.000152	0.016686	0.001277	0.001634	0.128591	0.022067

Table 1 Viscoelastic parameters of the PU foam from inverse.

The dynamic model of the transmissibility test rig for PU foam specimens is in equation (5), where M=116g is top mass, m_f=0.38g is foam mass, $u_d(t)$ is foam deformation and $u_b(t)$ is the base displacement. The transfer function (TF) of the system can be obtained from equation (5) using Fourier transform. Experimental TF curves using white noise and sine sweep are in Figure 2 (a) and show good agreement. The permeability of the foam is then obtained by inverse identification from the experimental TF curves (Figure 2 (b)). K increases almost linearly with the frequency: the pneumatic damping force is almost constant with frequency.

$$\left(M+\frac{m_f}{3}\right)\ddot{u}_d(t)+\frac{W^4\mu}{32\varphi KH}\dot{u}_d(t)+\frac{W^2}{H}\left[E_0u_d(t)-E_0\int_{-\infty}^{t}\sum_{i=1}^{N}a_ie^{-\alpha_i(t-\tau)}u_d(\tau)d\tau\right]=-\left(M+\frac{m_f}{2}\right)\ddot{u}_b(t) \qquad (5)$$

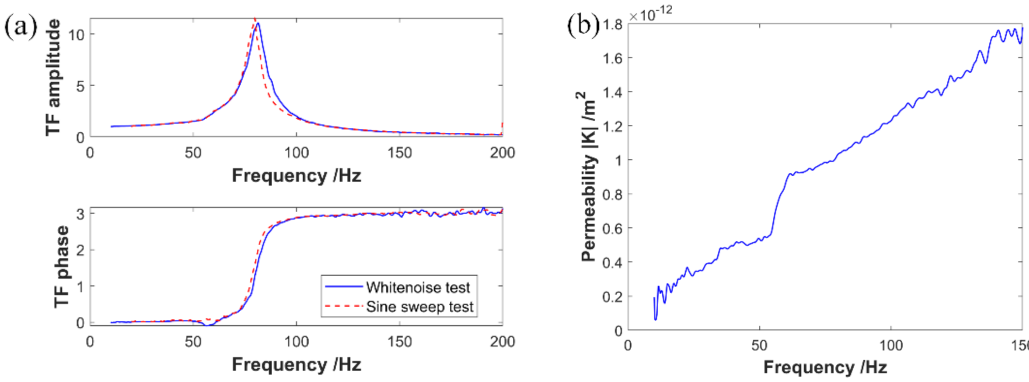

Figure 2 Transfer function of foam specimen in vibration test (a) and permeability identified (b)

3 Conclusions

A new dynamic poroelastic model for the vibration transmissibility of PU foams includes the pneumatic damping effect of the porosity, permeability and viscoelasticity of the PU material. Viscoelastic parameters are obtained by stress relaxation test and quasi-static cyclic loading test data simultaneously. The permeability of the foam at low frequencies is obtained by inverse identification from experiments.

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References

[1] M. Bianchi, and F. Scarpa. *Vibration transmissibility and damping behaviour for auxetic and conventional foams under linear and nonlinear regimes*. Smart Mater. Struct. 22.8 (2013): 084010.

[2] W. S. White, et al. *Experimental techniques and identification of nonlinear and viscoelastic properties of flexible polyurethane foam*. Nonlinear Dyn 22.3 (2000): 281-313.

[3] C. Tzikang. *Determining a Prony series for a viscoelastic material from time varying strain data*. (2000) NASA / TM-2000-210123.