Deterministic and statistical methods for the characterisation of poroelastic media from sound absorption measurements

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A framework is proposed for the characterisation of a rigid-backed poroelastic sample by using sound absorption measurements. The set of parameters of interest includes five transport parameters, two elastic properties, the mass density and the thickness of the sample. In the present work, all eight parameters are considered unknown and treated as random variables, whose nominal values and uncertainty ranges are of interest. The parameter estimation relies on model inversion, for which two complementary procedures are used, namely using deterministic and statistical tools. The methodology is applied to measurements in a two-microphone impedance tube.

1 Problem

The proposed approach aims at estimating the properties of poroelastic material samples from sound absorption measurements. The use of sound absorption coefficients provides experimental advantages with respect to measurements of the sound transmission behaviour. For instance, the measurement can be performed using a two-microphone impedance tube [1], and the rigid backing condition is relatively simple to set up. Despite the inherent simplicity and robustness of sound absorption measurements, these convey less information than the intrinsic acoustic properties of the sample such as the bulk modulus and equivalent density. The Biot-Johnson-Champoux-Allard model [2] for poroelastic media is here considered, which allows for the problem to be modelled analytically as a function of the eight unknowns.

2 Method

A recent method for the characterisation of dynamical systems [3, 4, 5] demonstrates the benefits of performing the model inversion in ascending frequency increments, as a series of sub-problems. This technique allows to gradually guide the parameter values towards the global solution of the problem, without terminating at local minima. In practice, each sub-problem is solved by means of a deterministic optimisation tool, such as the globally convergent method of moving asymptotes [6]. A statistical inversion approach is then used in order to refine the parameter estimates and obtain their uncertainty ranges. This is performed within the Bayesian framework using a Markov chain Monte Carlo approach [7].

3 Characterisation of a melamine foam sample

The present work applies the method to the experimental characterisation of poroelastic materials in a two-microphone impedance tube. Table 1 shows the obtained solution for a 24 mm-thick sample of melamine foam. The parameter estimates are provided as the maximum a posteriori estimate and their uncertainty ranges as the 90% credible intervals.

References

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Parameter	**Unit**	**Estimate**	**Uncertainty range**
Thickness	mm	23.14	23.01 ... 23.29
Mass density	kg·m^{-3}	11.958	11.562 ... 12.449
Porosity	-	0.9996	0.9906 ... 0.9998
Flow resistivity	kN·s·m^{-4}	18.709	18.516 ... 19.044
Tortuosity	-	1.001	1.000 ... 1.023
Viscous characteristic length	μm	70.93	69.09 ... 74.82
Thermal characteristic length	μm	159.07	155.92 ... 163.24
Young's modulus	kPa	196.697	190.276 ... 203.756
Viscoelastic damping ratio	%	4.696	4.364 ... 4.982

Table 1: Parameter values and uncertainty ranges obtained from the proposed approach for a sample of melamine foam.

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