High order X-FEM for the acoustic analysis of sound absorbing poro-elastic material with coupling interfaces

S.Wu[1], O. Dazel[2], G. Gabard[2], G.Legrain[1]

[1] École Centrale de Nantes, 1 rue de la Noë, 44321 Nantes, France
[2]Le Mans Université, 72085 Le Mans Cedex, France

Our work proposes a computational model based on the eXtended Finite Element Method (X-FEM) to predict the acoustic behaviour of porous structures coupled to other media. The poro-elastic material is modelled by a mixed formulation based on the Biot's thoery. This work demonstrates that the proposed method is able to alleviate the geometry limitation and to ensure the convergence of the Biot's equations simultaneously.

1 Background and problematic

Poro-elastic materials (PEMs) are widely used to build sound absorbers for practical noise control problem such as room acoustics ceiling and sound proofing in aircraft, cars and trains. Biot's thoery is one of common models to describe the energy absorption of the porous materials. In this work, we use a mixed formulation [1] based on the Biot's thoery as the model of the porous material. This formulation provides a full coupled description for the propagation of elastic solid and poro fluid waves, providing a natural coupling with other media.
Finite element methods (FEM) and wave based methods are two frequently used numerical methods for solving the Biot equations. The classical FEM suffers from convergence problems with respect to the number of degrees of freedom [2]. Wave based methods [3] show a better capacity to capture oscillation behaviour with less numbers of degrees of freedom. However, for most of wave based methods, extra operators at the interface are needed to couple all subdomains with different materials. Within the classical FEM, the mesh is needed to coincide with the geometry of the interface. If the mesh is not conforming at the interface, Lagrangian multiplier, penalty method and Nitsche's formulation are employed to weakly enforce the discontinuities between meshes as in wave based methods. All these requirements increase the computational cost especially when the methods are applied for complex geometry problems.

2 Proposed Method

To alleviate the geometrical limitation, the eXtended Finite Element Method (X-FEM) is taken into consideration. X-FEM was firstly proposed to solve crack growth problems without re-meshing [4] and was quickly applied to deal with material interfaces. In this work, we use a level-set function to locate the material interface implicitly. The elements split by the interface are enriched by a so-called ridge function [5] to introduce the weak discontinuity in the approximation inside the elements. Based on the idea of the X-FEM and the nature of the Biot equation, we provide the discretization strategies for two common coupling interfaces: acoustic-PEM and PEM-PEM. The Fields are approximated by high order Bernstein shape functions to ensure a proper rate of convergence for the Biot equation in high frequency.

3 Results and discussion

The sound pressure distribution in a car computed by the proposed method is given here. The driver seat is filled with XFM porous material modelled by the Biot formulation. An arbitrary normal velocity is imposed at the front windscreen under the frequency of 2000 Hz. A common situation where the driver's seat is switched from one position to another is simulated, which can be considered as a moving interface problem. The geometry of the problem and the mesh within our method are shown Figure 1. The profile of the driver seat is defined implicitly by level-set functions (in blue and red). Approximation of degree $p = 4$ is adopted to reach a converged solution using this mesh.
As seen in Figure 1, the discretization (meshes) is independent of the seat geometry as well as the position. The results in Figure 2 show that the variation of the acoustic pressure as the change of the seat position is well simulated and the absorption is captured in the region of the driver seat. This

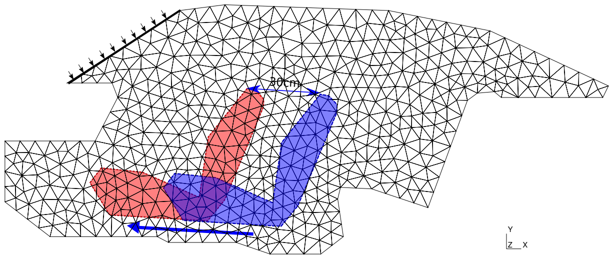
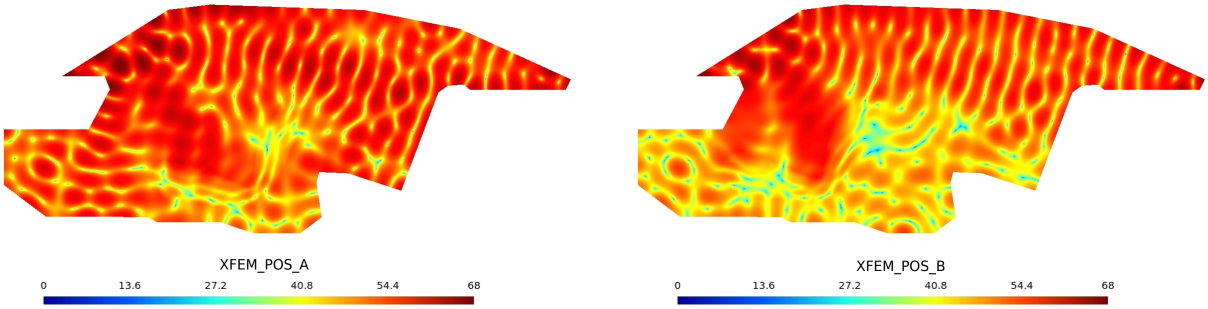


Figure 1: X-FEM mesh with level-set defined seat for stated problem



(a) (b)

Figure 2: Solution of the sound pressure (in dB) for position blue (left) and red (right)

example demonstrates the applicability and the computational convenience of the proposed method in a practical application.

Such a complex problem is solved using a relatively simple mesh. We believe that our method could be a competitive computational tool for analysing poro-elastic material involving complex geometries, even for the topological design of absorbing materials. More potential of the method is explored in the authors' current work such as "imperfect" medium interfaces behaving as strong discontinuities.

References

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