A non-locally reacting Impedance Model for Periodic Noise Control Treatments

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A Transfer Matrix (TM) based Noise Control Treatments (NCTs) modeling is proposed. Assuming each layer of the NCT to be periodic, a Generalized TM Method (GTMM) based on a Bloch modes basis reduction of the Finite Element (FE) model of the Unit Cell (UC) of each layer is employed to model the NCT in the wavenumber space and a inverse Discrete Fourier Transform (DFT) is exploited to build the impedance at the interface of the treatment with a fluid or a structure.

1 Introduction

Classical modeling techniques for NCTs are either based on detailed FE methodologies or simplified approaches. These last mainly rely on locally reacting impedance models obtained from testing or fast calculations performed by means of transfer matrix based analytical formulations assuming flat and homogeneous layers [1]. Although very efficient, this class of approaches may lack of accuracy for complex NCTs. To overcome this limitation, the in-plane heterogeneity of the treatment is taken into account by means of a GTMM [2] and the non-local behavior at discrete locations on the interface of the NCT is captured by means of an inverse DFT. The GTMM assumes each layer of the NCT to be periodic and, therefore, fully characterized by a UC. The latter is modeled by standard FE and the passage of Bloch waves, including the effect of the curvature, is imposed as boundary conditions. As a result, a TM can be obtained for each layer and then assembled to construct the TM of the NCT.

2 Theory

A NCT lying on the xy-plane has thickness h along the z-axis direction and is connected to a semi-infinite media (*e.g.* a fluid) or another system (*e.g.* a structure) through a rectangular surface of size $a \times b$ located at $z = h$. We are interested in the dynamic stiffness matrix, $\mathbf{D}$, of the NCT at the interface ($z = h$) when generic boundary conditions (*e.g.* semi-infinite fluid, hard wall, etc ...) are applied at the opposite side ($z = 0$). Assuming the NCT to be (i) periodic (of infinite extent) in the xy-plane, (ii) defined by its UC and (iii) subjected to an in-plane periodic motion with periods a and b, an inverse DFT can be used to map the NCT stiffness evaluated for a set of n discrete wavenumbers in the xy-plane on a lattice of n discrete locations over the interface area ($z = h$). In case of an unwrapped curved NCT, the wavenumber components, k_x and k_y, are defined on the curvilinear coordinates and the curvature is accounted for in the UC dynamics. In order to obtain a proper DFT operator, the wavenumber sampling vectors must have the following form

$$\mathbf{k}_x = \frac{2\pi}{a}\mathbf{O}_{\hat{M}_y} \otimes \left[\, -M_x \; \dots \; M_x \right] \;,\; \mathbf{k}_y = \frac{2\pi}{b}\left[\, -M_y \; \dots \; M_y \right] \otimes \mathbf{O}_{\hat{M}_x} \tag{1}$$

where $\otimes$ denotes the Kronecker product, $\mathbf{O}_n$ is a row vector of ones of size n, M_x and M_y are integers defining the maximum orders of wavenumbers considered, $\hat{M}_x = 2M_x + 1$ and $\hat{M}_y = 2M_y + 1$. The lattice has spacings $l_x = a/\hat{M}_x$ and $l_y = b/\hat{M}_y$. In case the dynamics of the NCT need to be projected on the Bloch modes, the interface must embrace an odd number of UCs and the wavenumber vectors must be enriched with the Bloch modes as

$$\mathscr{X} = \mathbf{k}_x \otimes \mathbf{O}_N + \mathbf{O}_M \otimes \mathbf{n}_x \;,\; \Upsilon = \mathbf{k}_y \otimes \mathbf{O}_N + \mathbf{O}_M \otimes \mathbf{n}_y \;, \tag{2}$$

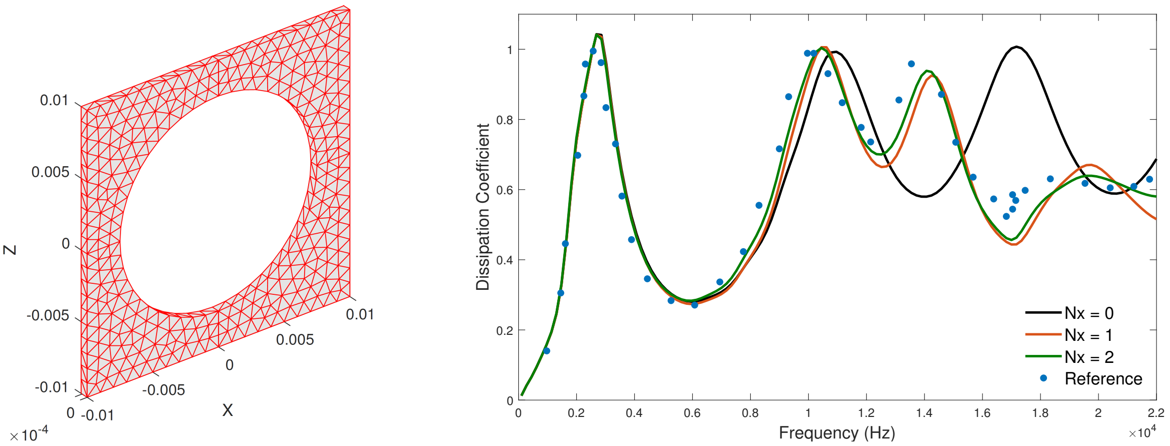
where $M = \hat{M}_x\hat{M}_y$ is the total number of cells, $N = \hat{N}_x\hat{N}_y$ is the total number of Bloch modes, $\hat{N}_x = 2N_x + 1$, $\hat{N}_y = 2N_y + 1$, N_x and N_y are the maximum orders of Bloch modes selected and the Bloch modes vectors are defined as

$$\mathbf{n}_x = \frac{2\pi}{l_x}\mathbf{O}_{\hat{N}_y} \otimes \left[\, -N_x \; \dots \; N_x \right] \;,\; \mathbf{n}_y = \frac{2\pi}{l_y}\left[\, -N_y \; \dots \; N_y \right] \otimes \mathbf{O}_{\hat{N}_x} \;. \tag{3}$$

The corresponding lattice coordinates are defined as $\mathbf{x} = \mathscr{X} a^2/(2\pi \hat{M}_x \hat{N}_x)$ and $\mathbf{y} = \Upsilon b^2/(2\pi \hat{M}_y \hat{N}_y)$. The dynamic problem is first solved in the wavenumbers space as $\hat{\mathbf{D}}\hat{\mathbf{q}} = \hat{\mathbf{F}}$, where $\hat{\mathbf{q}}$ and $\hat{\mathbf{F}}$ are the vectors of generalized displacement and forces collecting Bloch modes components and $\hat{\mathbf{D}}$ is a block diagonal matrix whose i-th block, $\hat{\mathbf{D}}^{(i)}$, is the stiffness matrix of size Nm related to the i-th couple of wavenumbers from vectors $\mathbf{k}_x$ and $\mathbf{k}_y$ and projected on the related Bloch modes and m is the number of degrees of freedom for each lattice node (1 for fluids, 3 for solids, 4 for porous). In case of a NCT defined as a generic arrangement of periodic layers, the TMs of all layers, $\hat{\mathbf{T}}_j^{(i)}$, must be first defined and the global TM, $\hat{\mathbf{T}}^{(i)}$, must be assembled as described in [2]. Then, the problem must be recast in stiffness form, thus obtaining $\hat{\mathbf{D}}^{(i)}$. The vectors of generalized displacements and forces at the lattice points can be expressed as $\mathbf{q} = \hat{\mathbf{S}}\hat{\mathbf{q}}$ and $\mathbf{F} = \hat{\mathbf{S}}\hat{\mathbf{F}}$ respectively. The transform map is defined as $\hat{\mathbf{S}} = \exp\left(-\mathrm{j}\mathbf{x}^T \mathscr{X} - \mathrm{j}\mathbf{y}^T \Upsilon\right) \otimes \mathbf{I}_m/\sqrt{NM}$, where $\exp(*)$ denotes the element wise exponential operator and $\mathbf{I}_n$ is the identity matrix of size n. Then, the stiffness matrix at the lattice points can be expressed as $\mathbf{D} = \hat{\mathbf{S}}\hat{\mathbf{D}}\hat{\mathbf{S}}^H$.

3 Application

The sound absorption of a 20 mm thick NCT made of foam with infinite cylindrical void inclusions and backed by a rigid wall (at $z = 0$) is considered. Inclusions are parallel to the y-axis, have diameter of 15 mm and are 20 mm spaced along the x-axis. The foam has properties: $\Phi = 0.95$, $\sigma = 8900$ Nsm^{-4}, $\alpha_\infty = 1.42$, $\Lambda = 180$ μm, $\Lambda' = 360$ μm, $E = 224$ kPa, $\nu = 0.4$, $\mu = 17$ %, $\rho = 6.1$ kgm^{-3}. It is modeled as an equivalent fluid according to the Johnson–Champoux–Allard rigid model [1]. The top fluid ($z > h$) has properties $c = 340$ ms^{-1} and $\rho = 1.284$ kgm^{-3}. The UC is modeled with the Tetra4 mesh depicted in Figure 1a. The acoustic impedance at the interface with the semi-infinite fluid is evaluated at the lattice points by assuming an infinite rigid baffle. Figure 1b presents the comparison of the dissipation coefficient at normal incidence predicted by the proposed approach with the result produced by a double porosity analytical model (Reference) [3]. It can be observed that Bloch modes are essential to capture the physics of the problem, especially above 17 kHz.



(a) Tetra4 mesh of the unit cell

(b) Dissipation coefficient at normal incidence

Figure 1: 20 mm thick NCT made of foam with cylindrical inclusions and backed by a rigid wall with in-plane dimension of 0.22 m × 0.22 m ($M_x = M_y = 10$, $N_y = 0$).

References

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